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Preference-Based Surrogate Modeling in Engineering Design

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Traditional techniques for validating and updating engineering models attempt to comprehensively improve model fidelity over the entire design space. Alternatively, this paper presents a preference-based strategic procedure for model building in simulation-based design optimization. It is based on the hypothesis that model validation over the entire design space will be time-consuming and may not be necessary, and a preferred approach is to validate them at potential optimal locations. On this basis, this paper introduces an integrated procedure that incorporates the statistics-based Kriging modeling method with an innovative preference-based approach to recursively build, validate, and update predictive surrogate models in the context of engineering design decisions. A distinguishing feature of this approach lies in its strategic investigation of model fidelity from the perspective of its relevance, usefulness, and completeness, thus maximizing its ability to find the most accurate results during design optimization while minimizing the computational cost involved in such model building. Two case studies, including a 21-bar truss design problem, are used as test-bed applications to illustrate the applicability of the preference-based modeling procedure, and the results are discussed from the perspectives of the efficiency of the overall design process and the accuracy of the resulting optimal design outcomes.

I. Introduction

AN ENGINEERING model is an approximation to reality, whose main purpose in simulation-based design optimization is to mimic system behavior for the purposes of engineering analysis and design. From an engineering design perspective, engineering models can be classified as descriptive models and predictive models [1]. A descriptive model aims to describe the reality as it is; it is typically built on principles of physics and validated against the observations made in carefully controlled experiments. As a representation of system behavior, a descriptive model is expected to be accurate, robust, and reliable over the entire design space and can be very useful in engineering analysis. On the other hand, a predictive model's primary purpose is to facilitate accurate and reliable search of optimality conditions during design optimization [2]. From a design optimization perspective, a predictive model is most useful if it can help to discriminate between competing design alternatives and lead to the identification of the optimal product design among them. Therefore, it is preferable to employ a model building approach that takes into account design optimality conditions. Unlike performance-based model building methods that aim to construct descriptive models using the entire design space, a preference-based modeling approach will be cost-effective and can still yield high-fidelity models. The process will require validations only in the select regions of optimality within the design space where the designer is most interested in evaluating competing design alternatives, and will naturally lead to high-fidelity models at those optimality regions. This process also reflects the intuitive design approach of an experienced designer, who would not pursue high-fidelity of the predictive model over the entire design space, but rather focus within the neighborhoods of the likely optimality scenarios. This paper presents the development of such an approach which can be used to build predictive models from a design optimization perspective.

The fundamental challenge to developing predictive models with high-fidelity at optimality conditions is in how to prune the design space strategically and selectively, and to identify optimality regions of interest for model validation, because such regions cannot be known a priori. This paper addresses the aforementioned challenge using well-established principles of engineering decision analysis. Here, predictive modeling is viewed as a decision-making process under uncertainty requiring tradeoffs between the cost of new information and the value of that new information [3]. Specifically, this approach recognizes the dilemma that the best predictive model cannot be built and validated unless and until the expected design outcomes are known; however, the best design outcomes cannot be known unless and until the perfect model is found. This is analogous to decision making under conditions of uncertainty, where perfect information about the future never exists. Therefore, it can be stated that the process of finding the best design can only be achieved through an iterative process of predictive model building and validating in the neighborhoods of such expected optima. Such a process will ensure that the conclusions obtained are consistent with not only the data used in generating the predictive model, but also with the design optimization results generated from the use of such models. On this basis, this paper presents an integrated methodology, using statistical techniques and decision analysis principles, for the development of predictive models in engineering design optimization.

In engineering optimization, when a large number of model simulations must be made, empirical response surfaces or surrogate models are often applied when the data gathering from simulation models are cost-prohibitive [4]. Recently, enormous advancements in numerical simulation tools have brought about high-fidelity engineering analysis models whose single simulation may take hours or days of supercomputer time [5–8]. Product optimization based on these simulation models becomes computationally very intense, which necessitates the use of surrogate models in design optimization. In this context, a surrogate model can be viewed as a simplified, analytically controllable, mathematical mapping that bridges design variables with product attributes. Essentially, it is the model of an analytical model or a metamodel, which has one more level of abstraction away from the reality compared with analytical models. The benefits of such a surrogate/metamodel can be significant in terms of the efficiency of optimization process, especially in the context of large-scale design problems involving multiple iterations. As a matter of fact, more and more surrogate models are being applied in lieu of actual expensive descriptive models in engineering design optimizations [6,8–10].

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Surrogate model building begins with initial data gathering. There are many different sampling schemes for exploring an entire design space effectively. Besides classic design of experiments (DOE) sampling methods, such as central composite, Box–Behnken, etc. [11,12], there are space-filling methods [10,13,14], such as Latin hypercube, orthogonal array, etc. [15–19]. DOE sampling methods are designed for physical experiments. Recognizing the random errors in the collected data, techniques such as blocking, replication, and randomization are used to minimize such random errors [11]. However, DOE sampling methods are not cost-effective for computer experiments where the collected data are deterministic [10,13,14]. Instead, for cost-effective exploration of design space through computer experiments, space-filling methods have been found to be more suitable [10,13–16]. Space-filling methods, as the name implies, spread sample points into an entire design space equally.

Irrespective of the choice of a particular method, a surrogate model can be constructed from an initial set of statistically-selected data and with relatively few expensive runs of the original analytical model. Depending on the uncertainty level associated with the collected data, the surrogate model is built as either an interpolation function or as a regression function. In general, the interpolation form of surrogate models can be appropriate and well suited if the actual model is of high-fidelity and the data collecting procedure is statistically well controlled [10]. Alternatively, if there is a significant randomness associated with the collected data, the least square regression methods are considered more suitable because of their noise filtering capabilities [20]. Popular surrogate modeling methods include least-squares fit regressions [4], spline functions [21], radial basic functions [22], neural networks [23], wavelets [24], fuzzy logic [25], etc. In addition, for representing nondeterministic numerical models, the prevalent method is Bayesian interpolation, also referred to as DACE (Design and Analysis of Computer Experiments) modeling in statistics literature [10,26], Gaussian process regression in the neural networks literature [27], and Kriging in the geostatistics society [28].

Irrespective of how the initial data is gathered and which surrogate modeling method is applied, the major challenge in surrogate modeling lies in constructing an accurate representative model from available information. With limited building data, a surrogate model is of inherently lower fidelity, thus necessitating sequential validation and updating work. Here, model validation can be achieved from a descriptive viewpoint such as in leave-k-out cross-validation, lack of fit test, etc. [29–34], where model fidelity is achieved through validation over the entire design space. Alternatively, cost-effective, high-fidelity models for design optimization can be achieved through validation of models at select optimality regions, thus eliminating the need for excessive computational efforts for high-fidelity over trivial design spaces away from optimality.

II. Preference-Based Surrogate Modeling

This paper views the central purpose of a surrogate model as a predictive model for the purpose of the cost-effective prediction of the best solution during design optimization. Accordingly, this paper presents the development of a preference-based surrogate modeling (PRESM) that enhances the traditional surrogate modeling by incorporating design optimization information in the modeling, assessing, and updating procedures. PRESM is a structured cost-effective approach to high-fidelity model building where the critical regions around the expected optimality locations are used to validate models. From a mathematical viewpoint, PRESM results in an optimization process through iterative engineering model building, similar to response surface methodology (RSM) [4], application-driven sequential design (ADSD) [35], and efficient global optimization (EGO) [36]. In RSM, a second-order polynomial is employed to make a least-squares fit regression to the sample data gathered through DOE sampling methods such as central composite design; the polynomial is then sequentially validated and updated at the predicted optimal points [4]. However, RSM is not suitable in multimodal scenarios. In ADSD, Kriging interpolation is applied and the cross-validation and jackknifing methods are used to select the updating point. At each validation stage, several candidates that are "halfway" from existing sample points are identified, and among them, the sample point associated with the estimated maximum jackknife variance is finally selected for validation [35]. EGO also employs the Kriging interpolation method, and here, the updating is done at new points selected based on some infilling sampling criteria, such as generalized expected improvement function, minimizing surprises, maximum variance, etc. [36,37]. PRESM also employs the Kriging interpolation method at the start of the model building process. However, what is unique in PRESM is that the design optimality information (preference-information) is explicitly incorporated into the model updating and validation process. Thus, this approach treats model fidelity not as a goal, but as a tool for facilitating engineering optimization. To this end, the method uses concepts from decision analysis to extract and mathematically represent design optimality information in the form of single- and multi-attribute utilities (SAUs and MAUs). It also uses the evolutionary genetic algorithm (GA) as the search process to locate several potential optimality candidates which then provide the basis for model evaluations and model updating. A schematic outline of the framework for PRESM is shown in Fig. 1. The four main steps are initial model building, preferential screening, preference-based model evaluation, and preference-based model updating.

A. Initial Model Building

The initial model building phase begins with the construction of a routine statistics-based performance model built using the Kriging method to mimic system input/output behavior based on the collected data at the selected sample points distributed over the entire design space. Note that this could easily be generated with a DOE sampling method or a space-filling method. Irrespective of the method employed and the sample size used, the resulting surrogate model can be expected to have distortions that can lead to erroneous optimal design outcomes. To achieve better fidelity, especially in the neighborhoods of optimality, a preference-based model updating is introduced in this paper. The preference model captures the design objectives and constraints in the form of an MAU, which then acts as a supercriterion for design optimization purposes. Thus, the setup enables models to be updated at the regions of design space of most interest from an optimality perspective, while avoiding unnecessary validations at trivial regions that are outside the design domain of interest. Details on the development of performance model and preference model follows.

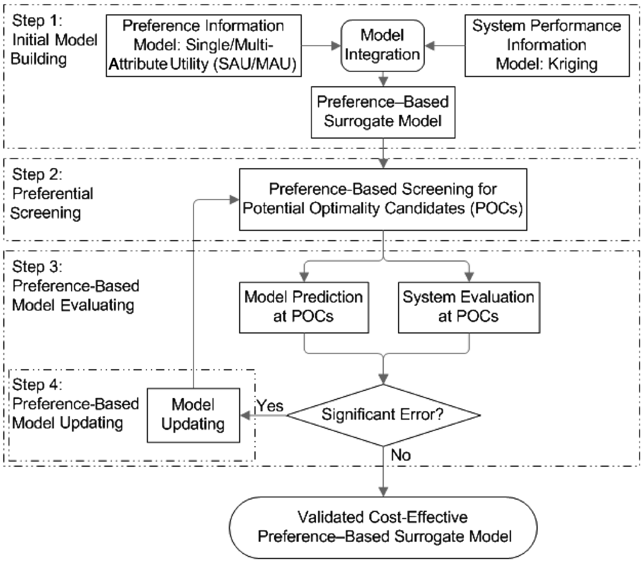


Fig. 1 Framework of preference-based surrogate modeling (PRESM).

1. Performance Model Building

The approach to performance model building is based on the Kriging method. In principle, Kriging assumes that prediction errors are the realization of a stochastic/Gaussian process with a zero mean and a nonzero variance and covariance [28,35,38]. It further assumes that the correlation between prediction errors can be modeled through spatial correlation functions (SCF) whose parameters are the maximum likelihood estimates based on the observed data [28]. As such, a Kriging model is a generalized linear regression model that takes the weighted linear combination of a set of collected data as its prediction model. These weights depend on the distances between sample points. They are used in such a way to ensure that the prediction at the previously sampled points will always remain at their observed response values and that the sampled points furthest away from unexplored locations have the least influence on the predicted values [28]. In recent times, the Kriging method has emerged as an effective tool for surrogate modeling in engineering applications [35,38,39]. This modeling method has gained acceptance as a legitimate design tool, and studies have shown that the Kriging method can be more accurate in terms of modeling a true function than lower-order, least-squares fit regression models when dealing with multimodal systems [39,40]. The general form of a Kriging model is described in Eq. (1):

$$y(\boldsymbol{x}) = f(\boldsymbol{x}) + Z(\boldsymbol{x}) \tag{1}$$

where $\boldsymbol{x}$ is the design variable vector, $y(\boldsymbol{x})$ is the actual response of the unknown system, $f(\boldsymbol{x})$ is a low-order polynomial fitted to the observed data through least-squares fit regression, and $Z(\boldsymbol{x})$ is the error between the prediction $f(\boldsymbol{x})$ and the true response $y(\boldsymbol{x})$. Here, $Z(\boldsymbol{x})$ is modeled as the realization of a stochastic/Gaussian process. Depending on the form and order of the polynomial $f(\boldsymbol{x})$ and the information used in estimating the regression coefficients, Kriging models can be classified into three types: ordinary Kriging models, universal Kriging models, and detrended Kriging models [39]. There is no difference between universal Kriging models and detrended Kriging models, if the correlation between any of the observed points is nil [39]. In this work, an ordinary Kriging model is employed which assumes a constant mean over the entire design space, i.e., $f(\boldsymbol{x}) = 1$. However, PRESM is equally applicable to other types of Kriging models, as well as any other method of performance model building.

2. Preference Model Building

Preference models, in the context of design optimization, refer to a quantitative mathematical representation of the optimality condition. It reflects the prioritization of objectives taking into account the value (utility) associated with the achievement of those objectives under conditions of uncertainty, because the final outcomes are yet unknown and have not yet materialized. In this paper, the well-established concepts from multi-attribute utility theory is used as the framework to efficiently extract and code the utilities associated with individual objectives and constraints, and likewise, to identify and generate the tradeoffs between multiple objectives in the system. Recent applications of utility theory in engineering design can be found in the book published by the American Society of Mechanical Engineers (ASME) [41]. Utility theory is a normative decision analysis approach with three main components, namely, options, expectations, and value, where the decision rule is that the preferred decision is that option whose expectation has the highest value (utility). It is based on the premise that the preference structure can be represented by real-valued functions and can provide a normative analytical method for obtaining the utility value ("desirability") of a design with the rule of "the larger the better" [42]. The major steps associated with this technique are [43] 1) identification of significant design attributes and the generation of design alternatives, 2) evaluation of SAU functions and tradeoff preferences, 3) aggregation of SAU functions into a system MAU function, and 4) selection of the alternative with the highest MAU value by rank ordering alternatives.

Here, the mechanism to get the preference structure is based on the notion of the lottery, referred to as a von Neumann–Morgenstern (vN-M) lottery, and from employing the certainty equivalent which is the value at which the decision maker is indifferent to a certainty option and a lottery between the best and the worst [42]. The lottery questions provide the basis for describing the logic between attribute bounds, where analytical function formulations are typically used to complete the preference structure description. Similarly, lottery questions form the basis for eliciting tradeoff information among attributes. In this representation, preference models are represented in an MAU function which then acts as the single supercriterion for subsequent design optimization. Specifically, an SAU function is built for each objective and constraint, and an MAU function is then constructed using a tradeoff approach [42,44].

a. Single Attribute Utility Function. In engineering design, the measure of degree about the goodness of design for each single objective can be modeled using an SAU function, where the design with the most optimal value for that objective will carry an SAU value of one, whereas the worst case scenario will have an SAU of zero. The SAU function is, thus, a normalized quantitative estimation to a designer's preference for that objective [42]. From a theoretical perspective, this also reflects the designer's logic as well as his/her expectation. The simplest SAU model is a linear model. This is shown in Eq. (2) and this form is used in the first case study. In Eq. (2), x is the attribute state value, SAU is the associated utility value, and m and b are the estimated coefficients.

$$\text{SAU}\,(x) = mx + b \tag{2}$$

In reality, however, the relationship between design outcome and its utility is not necessarily linear if one takes into account the risk associated with the expected outcomes. For example, the utility function will be concave if the designer is risk-averse, indicating that the designer considers the expected consequence to be more significant than the certainty equivalent. In other words, the risk-averse designer will prefer to behave conservatively, perhaps due to a lack of confidence or a fear of the consequence of a failure. In engineering design, due to safety considerations, utility assessments for certain objectives (attributes) such as stresses and deflection are likely to be treated as risk-averse in most design scenarios. Risk-reflecting utility functions are best represented using exponential utility functions [see Eq. (3)]. The three unknowns in Eq. (3), namely, a, b, and c, can be estimated by letting designers go through a vN-M test [42].

$$\text{SAU}\,(x) = ae^{bx} + c \tag{3}$$

As stated before, SAU functions are typically obtained by analyzing the designer's considerations of a set of lottery questions based on the concept of the certainty equivalent. This concept treats SAU as a monotonic function with the utility of $U_{\text{best}} = 1$ defined to be the most preferred value for an attribute function and $U_{\text{worst}} = 0$ for the least preference. SAU functions are then developed to describe the designer's compromise between the two extremes based on his/her priority-reflecting answers to the lottery questions. Specifically, the concept of certainty equivalent is used, which is regarded as a guaranteed result compared with the lottery between the two extreme values in which there is a probability p_0 of obtaining the best value and a probability of $1 - p_0$ of obtaining the worst value. A probability of $p_0 = 1$ causes the designer to choose the lottery, whereas a value of $p_0 = 0$ will lead to the selection of the certainty. That value of p_0 at which the designer is indifferent to the certainty and the lottery is characterized as the utility of the certainty. Using these three data points, the coefficients a, b, and c in Eq. (3) can be determined by simultaneously solving the three independent equations. The details of this procedure are demonstrated in the case studies.

b. Multi-Attribute Utility Function. The mathematical combination of the assessed SAU functions through the scaling constants yields the MAU function, which provides a utility function for the overall design with all attributes considered simultaneously. The scaling constants reflect the designer's preferences on the attributes, and they can be estimated based on the scaling constant lottery questions and the preference independence questions [42]. For the formulation of the MAU function, additive and multiplicative formulations are commonly considered. The main advantage of additive utility function is its relative simplicity, but the necessary assumptions can be restrictive. In addition, it is difficult to determine whether the requisite assumptions are reasonable in a specific engineering design problem. This is due to the fact that the assumptions are stated in terms of the preferences for probability distributions over consequences, with more than one attribute simultaneously varying. In addition, no interaction between the designer's preferences on various attributes is allowed. If additive independence assumption is not applicable, one may consider a multiplicative utility function, or a multilinear function. A typical multiplicative MAU function can be expressed as follows [42] and its development is shown in detail in the case study of a 21-bar truss design problem.

$$\prod_{i=1}^{n}(1 + Kk_i) = 1 + K \tag{4}$$

$$\mathrm{MAU}\,(x) = \left[\left(\prod_{i=1}^{n} Kk_i \mathrm{SAU}_i(x_i) + 1\right) - 1\right]\Big/K \tag{5}$$

c. MAU with Constraints. Historically, preferences on constraints are modeled as step functions, giving a value of "zero" for violation and "one" for nonviolation. In product design, however, a constraint need not always be absolute, and under certain situations, one may consider slight violations in lieu of a better design may be acceptable. Recognizing this, an SAU for constraint handling based on the Weibull function can be developed [42,45] which acts like a penalty function when there is a constraint violation. By quantifying the three points at the transition area that are associated with utility values 1, 0.5, and 0, the SAU of a constraint can be constructed in the form of Eq. (6):

$$\mathrm{SAU}\,(x) = e^{-(x/x_0)^m} \tag{6}$$

Thus, the constraints of the design can be captured and transformed into utility functions, while maintaining the integrity of the model. For multi-attribute scenarios, the effect of constraint violations in a multi-attributed system can be addressed by using the one-way relationship between attributes and constraints [45,46]. The general form of an MAU with constraints is shown as

$$\mathrm{MAU}(A, C) = [1 - \mathrm{SAU}(A^0, C^*)]\mathrm{SAU}(A, C^*)\mathrm{SAU}(C) + \mathrm{SAU}(A^0, C^*)\mathrm{SAU}(C) \tag{7}$$

where A stands for attributes, C stands for constraints, A^0 stands for the worst scenarios of A, and C^* represents feasible solutions where no constraints are violated. Equation (7) thus ensures that the utility of any alternative with a constraint violation will always be zero. Furthermore, it can be used to locate a region of feasibility before attempting to improve performance. Thus, preference models for multiple objective problems with constraints can be developed in a methodical manner, without loss of generality, and the supercriterion for design optimization can be achieved. This forms the basis for the construction of the preference-based surrogate models which is a mapping between the design variables and the MAU.

B. Preferential Screening

The second step in the PRESM framework is to prune the design space and find the special potential optimality candidates that can provide the most useful and relevant system information for subsequent model updating. In the context of design optimization, the best design candidates for model updating correspond to the locations that maximize the MAU value of the product design. As such, this can be achieved through the straightforward application of optimization techniques to the searching for the optimal design point associated with the maximum utility. For efficient and effective generations of optimal solutions, however, nongradient-based methods such as the GA [47,48] is preferred. The reason for the choice of a nongradient method is not only due to the fact that the overall utility functions may have a multimodal character, but also because surrogate models built using Kriging models or neural networks do not provide direct cost-effective gradient information.

In this study, the GA method is employed and PRESM leverages the parallel search functions of the GA to locate the potential optimality candidates by capturing the buried, transient, yet inherent data pattern in the artificial evolution of populations. Here, the uniformly distributed initial design alternatives are generated randomly and subsequently evaluated. Each member is then assigned a fitness value corresponding to its estimated MAU value. The higher this utility value is, the fitter the design alternative is, and the more likely it survives the evolution. The fitness value can mimic the utility value of MAU. And, in a manner similar to the lowest utility and the highest utility, the fitness values of zero and one can correspond to the worst and the best scenarios of possible outcomes. In this GA setup, the crossover and mutation techniques help to globally explore the design space, and the insertion technique ensures the continuous improvement of the average fitness of the entire population.

C. Preference-Based Model Evaluation

The GA helps to identify a set of potential optimality candidates in the preference-based surrogate model built by integrating a Kriging performance model with the preference models of the designer. The next step is the evaluation and validation of this model. For this, the predicted utilities at those potential optimal points are compared against actual utilities at the same locations using analytical models. There are many accepted ways of measuring the accuracy of a model, including the average percentage error method, which measures the average of the percent errors of the predictions against the true outputs of the system, and the root mean squared error (RMSE) method, which measures the model fidelity in terms of its variation and bias. In this paper, both the average percentage error method and RMSE method are employed for evaluation purposes.

D. Preference-Based Model Updating

As previously stated, potential optimality candidates serve as validation points in PRESM and the fidelity of the model is then checked through a comparison of the predicted utility values with the corresponding actual utility values at these candidate points. If the model fidelity is found to be not satisfactory at the screened optimality locations, the newly gathered data at these locations are used to update and further improve the model's fidelity in the neighborhoods of optimality. The design optimization process is then repeated using the updated model with the model fidelity error evaluation serving as the termination criteria, and a new set of potential optimality candidates is identified. The model updating process is continued until the error values are minimized and found to be within a preset level of acceptance. Thus, the PRESM model-building process facilitates the development of cost-effective, high-fidelity predictive models and the identification of the most optimal design outcomes, while ensuring its cost-effectiveness through the limited sampling in the neighborhoods of potential optimality locations for surrogate modeling and assessment.

A distinguishing feature of PRESM is that it does not pursue a comprehensive descriptive model, but views and updates predictive models as a means to identify the most desired design outcomes in an optimization process. Specifically, PRESM employs a preference-based updating strategy from a design decision perspective that focuses on the fidelity in the neighborhoods of optimality regions, while maintaining generality and cost-effectiveness that makes it equally applicable to any multi-objective constrained optimization problem.

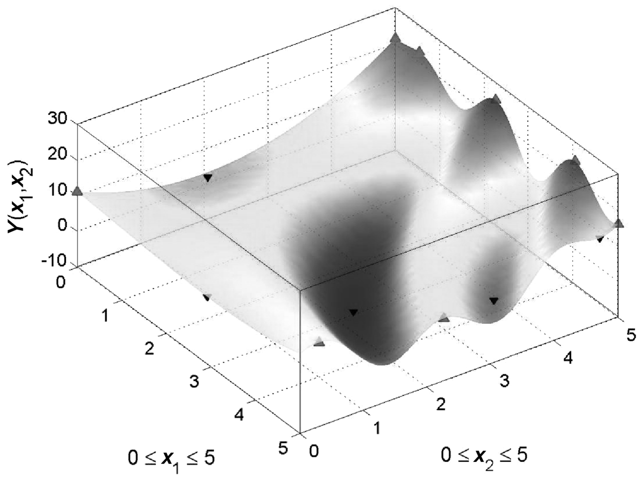


Fig. 2 Graphical representation of mystery function.

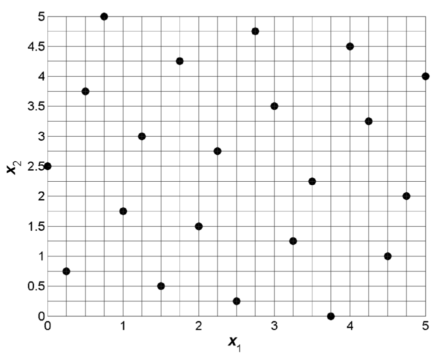


Fig. 3 Locations of initial 21 LHS points.

III. Case Studies

The application of PRESM is illustrated with the aid of the following two case studies. The first one is the unconstrained mathematical problem dealing with the maximization of the "mystery function" [see Fig. 2 and Eq. (8)], and the second one is a multi-objective constrained design optimization problem for a 21-bar truss (see the lists under Case Study 2). The implementation of PRESM is achieved in the numerical software MATLAB environment, and when necessary, toolboxes such as DACE [49] and GA are employed. The following subsections detail the case studies and discuss the experimental observations.

A. Case Study 1: Mystery Function

Figure 2 gives a graphical representation of the mystery function [Eq. (8)], which is chosen for its high nonlinearity with multiple local maximum points.

$$Y(x_1, x_2) = 2 + 0.01(x_2 - x_1^2)^2 + (1 - x_1) + 2(2 - x_2)^2 + 7\sin(0.5x_1)\sin(0.7x_1x_2) \quad \{x_1 \in [0,5] \cap x_2 \in [0,5]\} \tag{8}$$

To effectively explore the design space, an initial 21 sample points (see Fig. 3) designed by the Latin hypercube sampling (LHS) method is introduced. Table 1 lists the evaluation results at these 21 locations.

From the collected data, a Kriging model (see Fig. 4a) is constructed, which represents the initial preference model for the system. Simultaneously, the preference model is generated using principles from multi-attribute utility theory [42]. For this purpose, the test results in Table 1 are used, which indicates that the worst case scenario among the 21 sample sets is at point no. 15 where the objective value is at −4.17. On this basis, the value of −10 is considered as the hypothetical worst case scenario in the entire design space and is assigned a utility value of zero. This implies that for this maximization problem, the value of −10 is perhaps the least preferred option. Similarly, the possible best case scenario is estimated to be 30, which is higher than the observed maximum value of 21.71 at point no. 4 among the 21 sample sets, and its utility is assigned a value of one. For this mathematical problem, the preference model is created as a simple first-order linear polynomial function normalizing the performance values between −10 and 30 into the utility range between zero and one [Eq. (9)]. The resulting utility estimations at the initial 21 LHS points are calculated and shown in Table 2.

$$U = y/40 + 0.25 \tag{9}$$

After the integration of the performance model with the utility model, the initial preference-based surrogate model in the normalized utility scale is obtained (Fig. 4b). Because of the first-order linear character in the utility model, the obtained preference-based surrogate model is a linear transformation from its performance model.

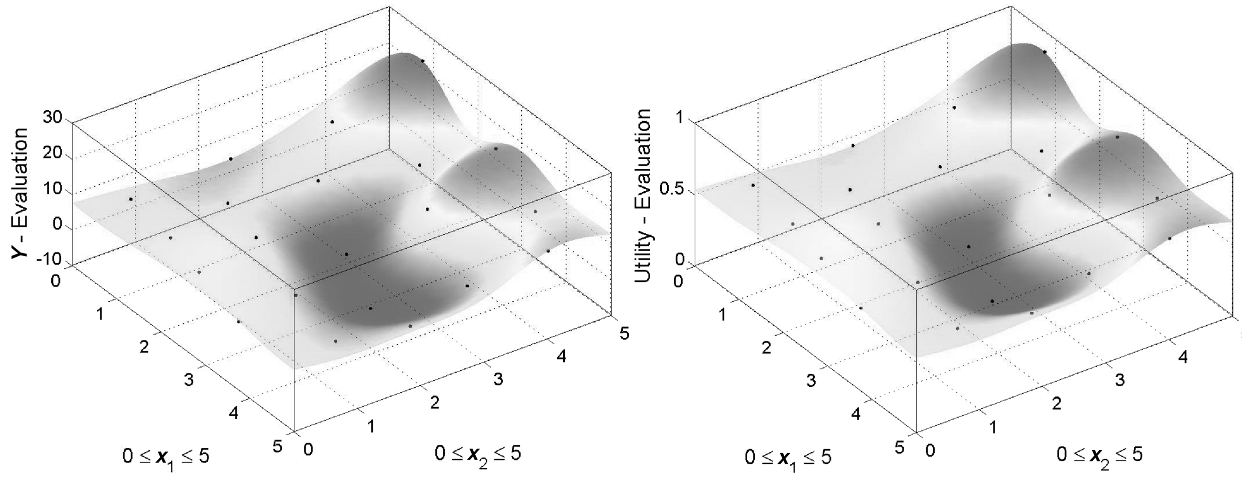
In step two, the preferential screening process is executed through applying the GA to the initial preference-based surrogate model. The

Table 1 Initial 21 LHS data

No.	x_1	x_2	Y	No.	x_1	x_2	Y	No.	x_1	x_2	Y
1	0.00	2.50	3.56	8	1.75	4.25	6.66	15	3.50	2.25	**−4.17**
2	0.25	0.75	5.99	9	2.00	1.50	6.65	16	3.75	0.00	9.23
3	0.50	3.75	10.42	10	2.25	2.75	−3.93	17	4.00	4.50	13.04
4	0.75	5.00	**21.71**	11	2.50	0.25	9.80	18	4.25	3.25	2.63
5	1.00	1.75	5.29	12	2.75	4.75	17.36	19	4.50	1.00	4.16
6	1.25	3.00	5.79	13	3.00	3.50	10.92	20	4.75	2.00	4.22
7	1.50	0.50	8.42	14	3.25	1.25	3.79	21	5.00	4.00	14.56

Table 2 Utilities of the original 21 LHS points

No.	x_1	x_2	U	No.	x_1	x_2	U	No.	x_1	x_2	U
1	0.00	2.50	0.34	8	1.75	4.25	0.42	15	3.50	2.25	**0.15**
2	0.25	0.75	0.40	9	2.00	1.50	0.42	16	3.75	0.00	0.48
3	0.50	3.75	0.51	10	2.25	2.75	0.15	17	4.00	4.50	0.58
4	0.75	5.00	**0.79**	11	2.50	0.25	0.49	18	4.25	3.25	0.32
5	1.00	1.75	0.38	12	2.75	4.75	0.68	19	4.50	1.00	0.35
6	1.25	3.00	0.39	13	3.00	3.50	0.52	20	4.75	2.00	0.36
7	1.50	0.50	0.46	14	3.25	1.25	0.34	21	5.00	4.00	0.61



a) Initial Kriging model

b) Initial preference-based surrogate model

Fig. 4 Initial surrogate models of mystery function.

potential optimality candidates are then identified at the locations with the highest utility values. In the first trial, the observation indicates five unique potential optimality candidates (Table 3). In step three, the evaluations of the model fidelity at these points are carried out. Table 3 lists the validation results: the RMSE is 0.1, the average error is 12.7%, the maximum absolute error is 0.155, and the maximum relative error is 24.9%. As one would suspect, these errors are significant, confirming that an initial model built without any regard to actual design can and will often end up with a low-fidelity model. Therefore, model evaluations are not only important to check the appropriateness of the design outcomes, but can also be the basis for improving the model's fidelity. Accordingly, step four, the model updating step, begins by adding the potential optimality candidates to the data pool used in the initial model building.

This augmented I/O data is then used to build an updated model and the modeling process is repeated. The performance values of the new data sets are found to be within the bounds of the hypothetical extreme scenarios, namely, $(-10, 30)$. Therefore, the utility model is kept unchanged. The process of locating the new potential optimality candidates, evaluating and validating the model at these locations, and updating the model as necessary based on the augmented samples continues until the fidelity of the model reaches the specified acceptable level. The results of the next four iterations of the PRESM are listed in Table 3. Figure 5 shows the error convergence history. Figure 6 shows the progression of the preference-based surrogate model in the iteration process of PRESM. Both Figs. 5 and 6 indicate that the model fidelity improves iteratively and preferentially at the regions of most interest from a design optimization perspective.

For comparison purposes, traditional surrogate-based optimizations are conducted. To ensure generality, 30 Kriging models are built on randomly selected 30 LHS sampling schemes, respectively, each of them having 34 sample points. Subsequently, optimizations are carried out using these 30 surrogate models for finding the maximum value of the mystery function. Table 4 lists the optimization results. The predicted global optimal point locations demonstrate randomness, with no clear indication of the actual maximum point or the value. Illustratively, Fig. 7 highlights that the optimal predictions (dot marks) made through statistical models alone are less likely to be reliable and accurate (triangle mark indicates the true maximum). Quantitatively, it is found that there is only a very small possibility, 5 out of 30, that the results will be accurate (bold values in Table 4). In summary, traditional surrogate-based optimizations can often produce optimal solutions with significant uncertainty, and model assessment and validation from a design optimization perspective can significantly enhance the model's fidelity in a cost-effective manner and lead to reliable and accurate design outcomes.

B. Case Study 2: 21-Bar Truss Problem

The application of PRESM to an engineering design problem is demonstrated through the 21-bar truss problem with seven design variables (Fig. 8). The 3-m-high and 12-m-long truss structure is constructed of straight and uniform steel bars pinned together. It is fixed at its left support and simply supported at its right end. A downward force $F = 100,000$ N (equivalent to a 10 ton object site on the truss) is acting at the low-midjoint of the truss. In this case, the design objectives are to simultaneously minimize both the weight W of the truss and the maximum vertical deflection D_M of the system, while at the same time ensuring that the structure is stable. This requires that both the tensile stress and the compression stress within each bar member are less than the allowed nominal stresses, respectively, and that the compression stress in each bar member is less than the critical buckling stress associated with that bar member.

Table 3 Potential optimality candidates at each stage for linear maximizing preference

Iteration	x_1	x_2	Actual U	Predicted U	Absolute error	Relative error	RMSE	Average error
Stage 1	3.02	4.78	0.630	0.785	**0.155**	24.60%	0.100	12.7%
	3.51	4.74	0.589	0.736	0.147	**24.90%**		
	4.79	4.94	0.720	0.664	−0.056	−7.80%		
	0.98	4.91	0.787	0.813	0.026	3.30%		
	0.37	4.94	0.846	0.821	−0.025	−2.90%		
Stage 2	2.52	4.92	0.860	0.817	**−0.042**	−4.90%	0.031	3.6%
	4.96	4.70	0.695	0.730	0.035	**5.00%**		
	0.81	4.49	0.744	0.779	0.035	4.70%		
	0.37	4.77	0.808	0.835	0.027	3.30%		
Stage 3	2.22	4.87	0.885	0.848	**−0.037**	**−4.10%**	0.017	1.0%
	0.57	4.92	0.850	0.858	0.008	0.90%		
Stage 4	2.38	4.78	0.870	0.888	**0.018**	**2.10%**	0.008	0.4%
Stage 5	2.38	4.71	0.856	0.854	**−0.002**	**−0.20%**	0.001	0.0%

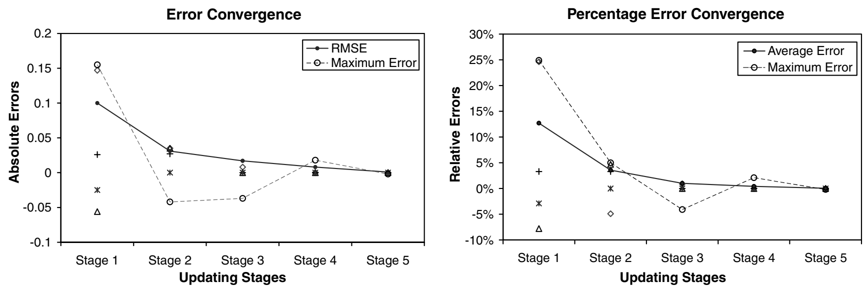
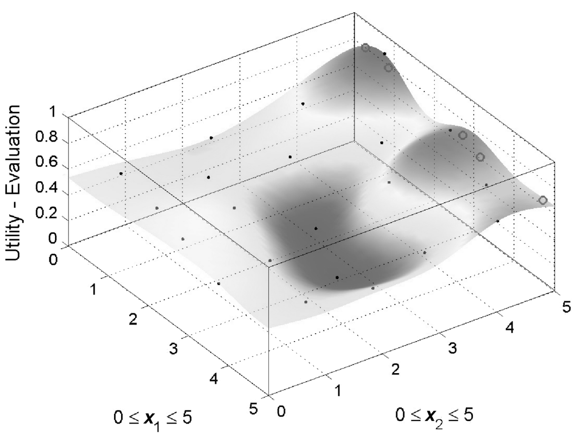
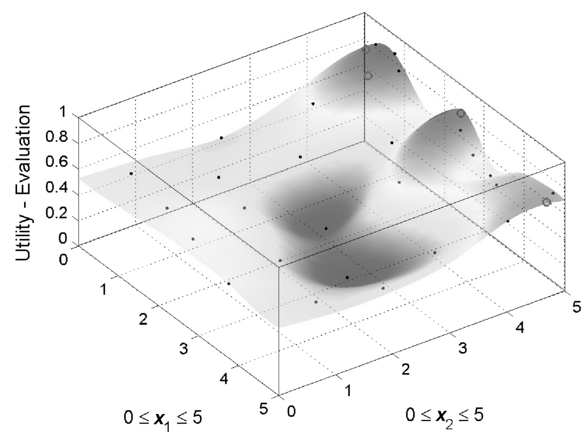


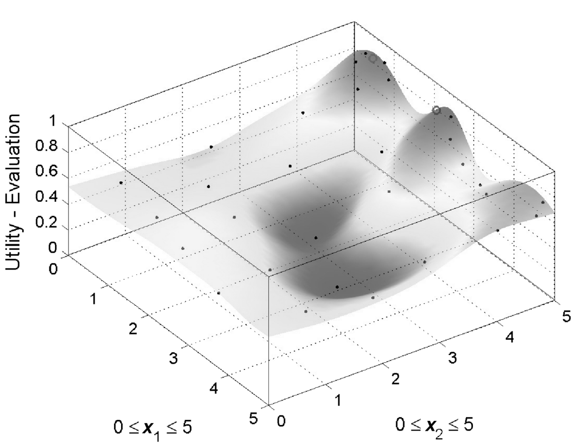
Fig. 5 Error convergences along with iterations.



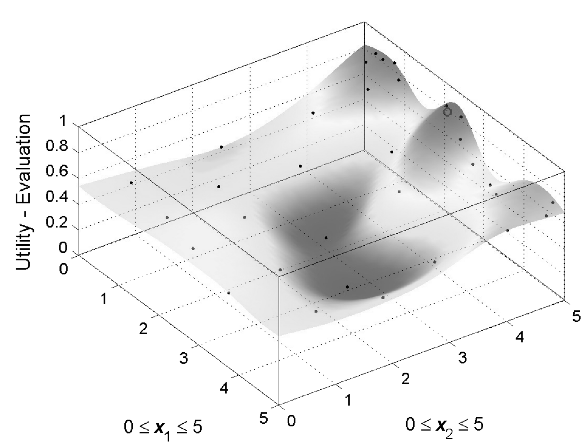
a) Initial Kriging model



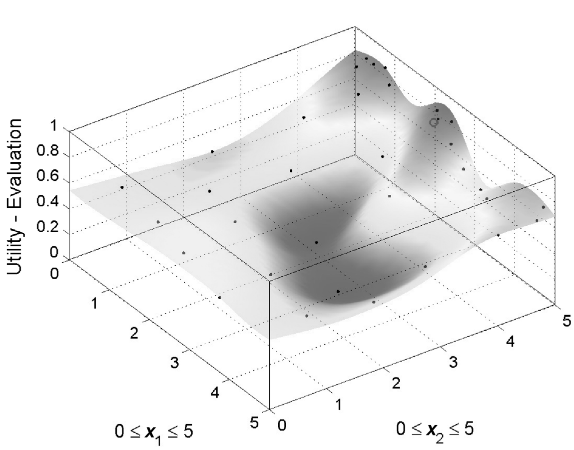
b) Kriging model at the second stage



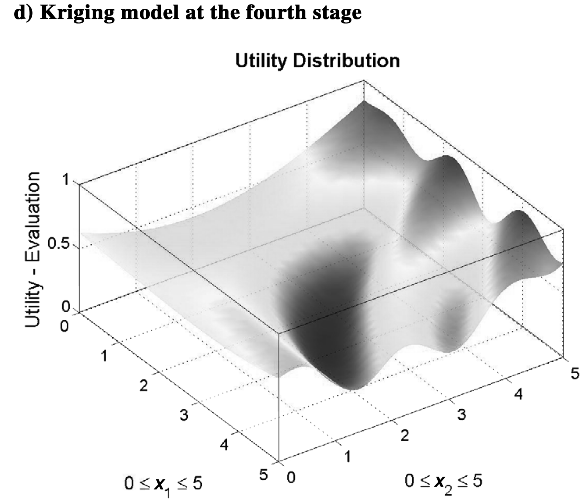
c) Kriging model at the third stage



d) Kriging model at the fourth stage



e) Final Kriging model



f) Actual model of utililty

Fig. 6 Progression of Kriging models.

Table 4 Optimization results from 30 different LHS schemes

No.	x_1	x_2	Y	No.	x_1	x_2	Y	No.	x_1	x_2	Y
1	4.05	5.00	22.91	11	**2.41**	**4.81**	**23.17**	21	4.69	4.57	20.45
2	3.04	4.81	31.94	12	4.53	4.77	18.10	22	4.96	4.51	21.77
3	3.86	4.55	23.02	13	2.58	4.50	18.63	23	**2.30**	**5.00**	**24.92**
4	0.26	4.93	19.55	14	4.45	4.77	20.36	24	2.62	4.64	17.80
5	0.66	4.90	19.85	15	4.71	5.00	19.87	25	0.35	4.90	19.93
6	2.95	4.42	21.82	16	4.45	4.99	20.77	26	0.47	4.84	20.83
7	**2.40**	**4.85**	**20.23**	17	2.59	4.97	21.58	27	**2.33**	**4.95**	**22.50**
8	2.53	5.00	22.25	18	2.47	4.66	18.38	28	0.58	5.00	22.03
9	4.79	5.00	28.89	19	0.58	4.95	20.49	29	0.08	4.94	20.25
10	**2.38**	**4.96**	**23.30**	20	4.37	4.72	21.41	30	3.43	4.88	24.82

For simplicity, it is assumed that the cross-sectional area of all truss bar members are constant (100 cm^2). Taking into consideration the symmetry in the truss structure and boundary conditions, there are seven design variables, namely, the horizontal positions of joints one and two $(x1, x2)$, the locations of joints three and four $(x3, y3, x4, y4)$, and the vertical position of joint six $(y6)$. Because the failure in strength of any bar member is considered the failure of the entire truss structure, only the maximum tensile stress σ_{TM} and the maximum compression stress σ_{CM} among all truss members are explicitly considered as constraints. In addition, in order for the truss to be stable, the minimum ratio R_{BK} of stress σ over the associated critical buckling stress must be greater than -1, so that there will be no truss member whose inner compression stress is bigger than its critical buckling stress. The design problem can then be formulated as a two-objective optimization problem with three constraints and seven design variables:

Minimize
1) total weight W
2) maximum vertical displacement D_M
Subject to
1) Maximum tensile stress $\sigma_{TM} < 200$ MPa.
2) Maximum compression stress $\sigma_{CM} > -200$ MPa.
3) Minimum ratio $R_{BK-MIN} > -1$.
4) $0.8 \text{ m} < x_1 < 3.0$ m.
5) $3.3 \text{ m} < x_2 < 5.5$ m.
6) $0.8 \text{ m} < x_3 < 3.0$ m.
7) $0.8 \text{ m} < y_3 < 3.0$ m.
8) $3.3 \text{ m} < x_4 < 5.5$ m.
9) $0.8 \text{ m} < y_4 < 3.0$ m.
10) $0.8 \text{ m} < y_6 < 3.0$ m.

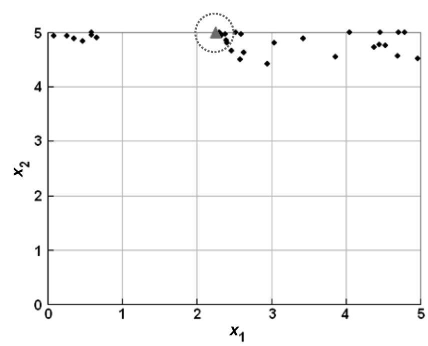


Fig. 7 Optimization results from 30 different LHS schemes.

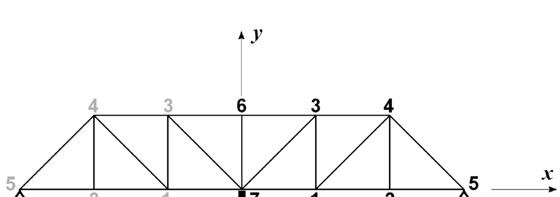
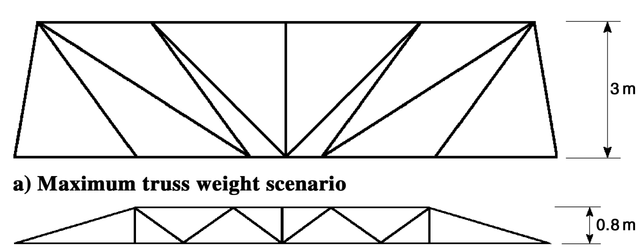


Fig. 8 Twenty-one-bar truss problem.

The ratio $R_{BK-MIN} = \min(\sigma_i / S_{cr-i})$, in which $i = 1, 2, \ldots, 21$, and σ_i is the axial stress of the ith truss member whose critical buckling stress $S_{cr-i} = \pi^2 E / (L_i/\rho)^2$ [50]. In this formula, E is the Young's modulus of the truss material, ρ is the radius of gyration of truss members, and L_i is the length of the ith truss member.

Similar to the procedure outlined in the first case study, a set of 70 initial sample points is selected based on the LHS method integrated with the maximin criteria. The results at these initial 70 sample points are listed in Table A1 in the Appendix. They are collected through the execution of finite element models of the truss structure using ANSYS [51].

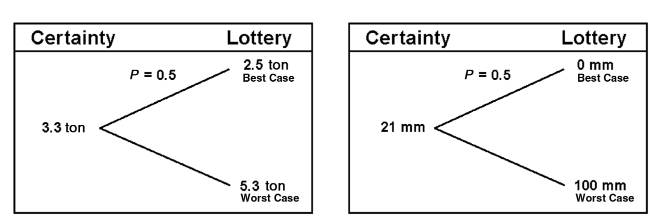
From the collected information at the initial 70 LHS points, the performance models of the objectives and constraints are generated. They are represented in the form of ordinary Kriging models. Simultaneously, the corresponding preference models are constructed through the applications of utility theory. For example, for estimating the utility model of attribute W, a worst case scenario of 5.3 ton is estimated and is assigned a utility value of zero. Similarly, the best case scenario of 2.5 ton is assigned the value of one. Figure 9 shows the truss configurations corresponding to the



a) Maximum truss weight scenario

b) Minimum truss weight scenario

Fig. 9 Truss configurations associated with both maximum and minimum weights.



a) b)

Fig. 10 Lotteries for a) weight, b) displacement.

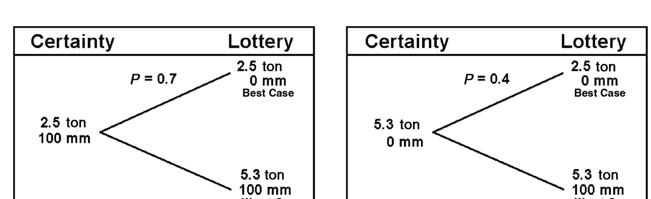


Fig. 11 Weight and displacement tradeoff lotteries.

Table 5 Stages of PRESM for the 21-bar truss problem

Iteration	x_1	x_2	x_3	y_3	x_4	y_4	y_6	Actual MAU	Predicted MAU	Absolute error	Relative error	RMSE	Average error
Stage 1	2.368	4.366	2.063	1.396	3.325	1.258	1.790	0.714	0.764	0.050	7.03%	0.048	6.61%
	2.334	4.650	1.917	1.382	3.687	1.206	1.725	0.714	0.756	0.043	5.99%		
	2.213	4.471	1.998	1.372	3.317	1.318	1.566	0.711	0.756	0.045	6.37%		
	2.358	4.672	2.179	1.543	3.317	1.298	1.790	0.688	0.755	**0.067**	**9.78%**		
	2.200	4.484	1.830	1.298	3.311	1.211	1.636	0.726	0.754	0.028	3.85%		
Stage 2	2.215	4.454	1.855	1.273	3.672	1.245	1.586	0.724	0.733	0.009	1.24%	0.014	1.68%
	2.198	4.455	1.827	1.217	3.444	1.255	1.554	0.729	0.733	0.003	0.47%		
	2.174	4.465	1.989	1.404	3.858	1.258	1.520	0.708	0.730	**0.023**	**3.19%**		
	2.200	4.436	1.862	1.104	4.323	1.254	1.505	0.710	0.730	0.019	2.68%		
	2.197	4.436	1.974	0.944	3.579	1.258	1.720	0.723	0.729	0.006	0.83%		
Stage 3	2.364	3.314	1.505	0.820	3.302	0.808	1.310	0.826	0.830	0.004	0.52%	0.008	0.88%
	2.313	3.453	1.299	0.820	3.302	0.808	1.426	0.816	0.828	**0.012**	**1.49%**		
	2.313	3.401	1.560	0.820	3.323	0.909	0.935	0.827	0.818	−0.009	−1.05%		
	2.060	3.689	1.240	0.830	3.302	0.808	1.446	0.808	0.818	0.010	1.25%		
	2.497	3.585	1.849	0.820	3.302	0.808	1.444	0.816	0.817	0.001	0.08%		
Stage 4	2.364	3.340	1.564	0.880	3.384	0.802	0.973	0.833	0.832	−0.001	−0.10%	0.004	0.44%
	2.190	3.615	1.249	0.839	3.341	0.839	0.876	0.832	0.829	−0.006	−0.69%		
	2.362	3.649	1.780	0.812	3.363	0.902	0.979	0.820	0.822	0.002	0.20%		
	2.490	3.405	1.799	0.839	3.345	0.959	0.965	0.817	0.821	0.004	0.46%		
	2.246	3.309	1.740	0.969	3.363	0.902	0.979	0.815	0.821	**0.006**	**0.74%**		

maximum weight scenario (Fig. 9a) and the minimum weight scenario (Fig. 9b), respectively.

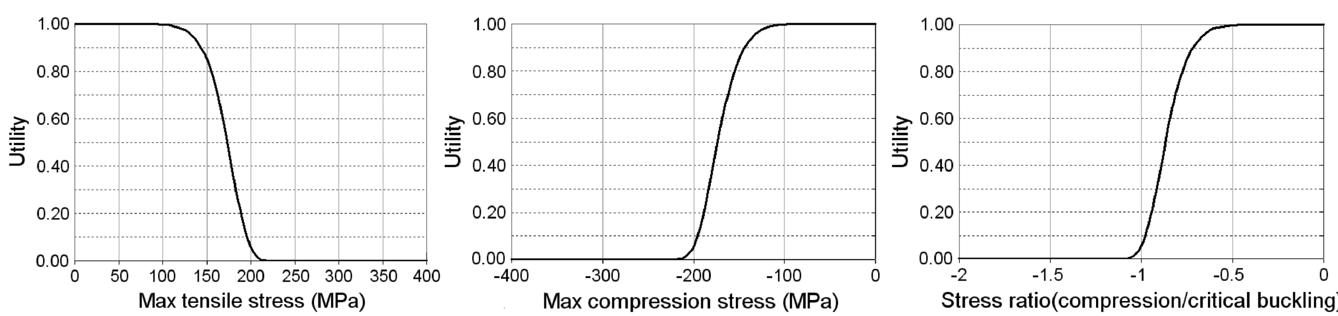
The utility model of weight W is then obtained through a vN-M lottery test. Figure 10a shows the lottery test for finding the SAU of W. Here, the designer is indifferent to the certain truss weight of 3.3 ton and the likelihood of 50% toward the best value of $W = 2.5$ ton. This results in a utility model for W:

$$\text{SAU}\,(W) = 6.74e^{-0.703W} - 0.162 \tag{10}$$

Similarly, the lottery question about the maximum vertical displacement is set up as shown in Fig. 10b and the utility model of it is estimated as

$$\begin{cases} \text{SAU}(D_M) = 1.06e^{-0.03D_M} - 0.06 & \\ \text{SAU}(D_M) = 0 & \text{if } D_M > 100 \text{ mm} \end{cases} \tag{11}$$

To simultaneously minimize both the truss weight W and the maximum vertical displacement D_M, an optimal tradeoff must be made because these two objectives are actually in conflict with each other. When the truss weight W is reduced, the rigidity of the structure may be compromised leading to increased displacement D_M, whereas by reducing the displacement D_M, the rigidity of the structure needs to be increased, which requires increasing the weight W. A multi-objective model that relates these two conflicting objectives is constructed by solving for the coefficients k_i in the standard MAU expressions [Eqs. (4) and (5)] through the applications of two vN-M lottery tests (Fig. 11). In the lotteries, the best scenario is the combination of all the best attribute values and the



a) Tensile stress **b) Compression stress** **c) Ratio of buckling safety**

Fig. 12 SAUs of stress constraints.

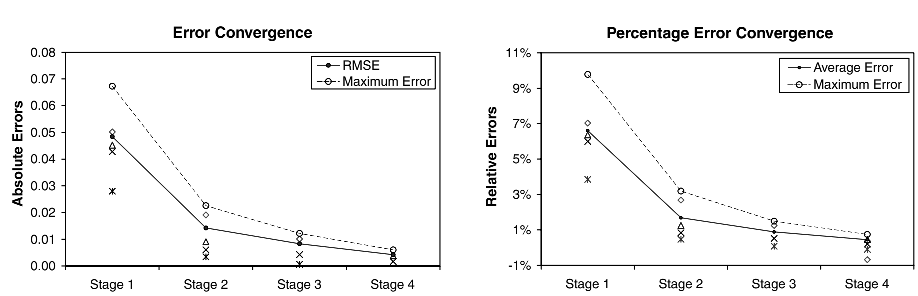
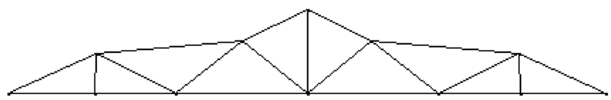
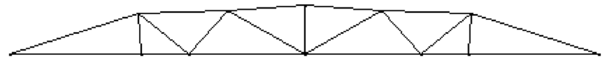


Fig. 13 MAU error convergences during iterations.



a) Predicted by traditional surrogate-based optimization



b) Predicted by implementing PRESM framework

Fig. 14 Comparison of predicted best truss designs.

worst scenario is a combination of all the worst ones. To quantify k_i, a mediocre certainty option is set up as the combination of the best value of the ith attribute and the worst values of all the other attributes. For example, as shown in Fig. 11, the coefficient k is 0.7 for attribute W, because the designer is indifferent to the certain situation (2.5 ton, 100 mm) and a 70% likelihood of the best case scenario (2.5 ton, 0 mm). Similarly, the coefficient k is 0.4 for attribute D_M, because the designer is indifferent to the certain value (5.3 ton, 0 mm) and a 40% likelihood of the best scenario (2.5 ton, 0 mm).

Substituting the coefficient values for k_i in Eqs. (4) and (5) results in

$$(1 + 0.7K)(1 + 0.4K) = 1 + K \tag{12}$$

$$K = -0.36 \tag{13}$$

$$\mathrm{MAU}\,(W, D_M) = \frac{[1 - .25\mathrm{SAU}(W)][1 - .14\mathrm{SAU}(D_M)]}{-0.36} + \frac{1}{0.36} \tag{14}$$

In a similar manner, the utility models for the three stress-related constraints can be estimated and modeled as Eqs. (15–17), respectively. Their characters are shown in Fig. 12.

$$\mathrm{SAU}_{\mathrm{TM}} = e^{-(\sigma_{\mathrm{TM}}/180)^{10}} \tag{15}$$

$$\mathrm{SA\,U}_{\mathrm{CM}} = e^{-(-\sigma_{\mathrm{CM}}/180)^{10}} \tag{16}$$

$$\mathrm{SA\,U}_{\mathrm{BK-MIN}} = e^{-(-R_{\mathrm{BK-MIN}}/0.9)^{10}} \tag{17}$$

This completes the attribute models for all the objectives and constraints, and the MAU model, which represents the preference model for the system, can be formulated as follows:

$$\mathrm{MAU} = \mathrm{MAU}(W, D_M)\mathrm{SAU}_{\mathrm{TM}}\mathrm{SAU}_{\mathrm{CM}}\mathrm{SAU}_{\mathrm{BK-MIN}} \tag{18}$$

The original two-objective constrained optimization problem is thus reduced to an unconstrained optimization of the supercriterion problem involving the MAU. The MAU is then integrated with Kriging performance models to form the preference-based surrogate model.

The remaining steps are similar to the first case study. As before, the initial preference-based surrogate model is screened using the GA for locating the optimality locations of the highest utility value. This results in five unique optimal points (Table 5), where the bold values indicate the potential optimality candidate (POC) (see Fig. 1) is associated with the maximum error among the sets of POCs. Subsequently, preference-based model evaluation and validation is conducted at these locations. Table 5 lists the experimental results for four iterations and Fig. 13 shows the corresponding error convergence history. The results appear to indicate that there are fluctuations in the errors at specific locations during iteration, while there is still a trend toward general convergence of the average errors. This is to be expected when dealing with highly nonlinear problems with potentially multiple local optimal points, where sample sizes may not be able to sufficiently and accurately capture all the important system features. Nevertheless, the PRESM process still leads to error convergence within a few iterations, indicating enhancement in the model fidelity and accuracy in resulting optimal solutions.

Once again, for comparison purposes, the traditional surrogate-based optimization is carried out using a surrogate model of the identical Kriging type built from 90 LHS sample points (See Table A2 in the Appendix). The predicted optimal design solutions from both methods are compared in Table 6 and Fig. 14, showing the different truss configurations. Figure 14a is the predicted optimal result from the Kriging model (built on 90 sample points) without updating, which has an MAU value of 0.760, whereas Fig. 14b is the result from PRESM application at the same cost, i.e., 90 sample points, which has an MAU value of 0.833. Though this is just one comparison, it nonetheless indicates that the PRESM can improve the resulting solutions, as one would expect the addition of sample points in the neighborhood of optimality to yield better solutions than otherwise.

IV. Summary

This paper presents PRESM, a novel preference-based surrogate modeling procedure, for use in engineering design optimization. PRESM is built on the hypothesis that the purpose of models in design optimization is to facilitate the identification of reliable and accurate optimal design outcomes. Accordingly, the method studies the surrogate model's relevance, usefulness, and completeness from a design decision perspective, and selectively uses new information obtained in potential optimal regions recursively to assess, update, and enhance surrogate model fidelity. The salient features of this integrated approach include the building of performance models using the Kriging method, the construction of preference models using multi-objective attribute utility functions, and a cost-effective assessment of model fidelity at potential optimal design points through a preferential screening process based on a genetic algorithm search. The results from the case studies indicate that the implementation of PRESM facilitates the enhancement of surrogate model fidelity in a cost-effective manner by screening the design space and validating the model and the corresponding model-based design outcomes in the neighborhoods of the optimality regions. The case study with the 21-bar truss problem also highlights the applicability of PRESM to multi-objective constrained design optimization problems.

Table 6 Details of the predicted results

	Design variables							Attributes		Constraints			Utility
	x_1	x_2	x_3	y_3	x_4	y_4	y_6	W	D_M	σ_{TM}	σ_{CM}	$R_{\mathrm{BK-MIN}}$	MAU
No.	m	m	m	m	m	m	m	ton	mm	MPa	MPa	——	——
a	2.639	4.268	1.295	1.061	4.238	0.800	1.689	2.95	3.96	22.17	−19.74	−0.099	0.760
b	2.364	3.340	1.564	0.880	3.384	0.802	0.973	2.73	5.78	25.20	−30.88	−0.081	0.833

Appendix: Sample Data Sets

Table A1 Initial 70 LHS sample data sets

	Design variables							Attributes		Constraints		
	x_1	x_2	x_3	y_3	x_4	y_4	y_6	W	D_M	σ_{TM}	σ_{CM}	$R_{\mathrm{BK-MIN}}$
No.	m	m	m	m	m	m	m	ton	mm	MPa	MPa	——
1	2.356	3.473	1.821	0.879	3.441	2.921	1.350	3.717	5.5	23.77	−23.16	−0.082
2	1.601	4.039	1.413	1.979	5.484	0.879	1.287	3.626	3.6	20.17	-25.96	−0.134
3	2.387	3.881	0.973	1.759	4.981	2.953	1.633	4.074	2.0	14.29	−18.52	−0.096
4	1.633	4.573	2.607	2.041	5.201	2.104	2.513	4.106	1.5	10.36	−12.13	−0.054
5	0.941	4.479	1.853	2.073	5.233	0.816	2.481	3.756	1.2	10.00	−12.38	−0.093
6	2.607	3.630	2.324	1.413	3.410	1.790	1.853	3.313	2.0	13.01	−16.48	−0.059
7	2.041	4.321	1.350	2.544	4.416	2.607	1.161	4.047	8.7	35.63	−52.94	−0.088
8	1.444	4.824	1.287	1.381	4.887	2.356	1.696	3.744	3.3	17.06	−18.21	−0.147
9	1.161	3.504	2.890	1.161	4.290	1.193	1.476	3.558	3.7	15.42	−21.58	−0.110
10	2.136	3.724	1.570	2.984	5.139	2.450	2.136	4.338	2.0	14.22	−15.96	−0.056
11	2.953	5.201	2.796	1.350	4.196	1.759	1.004	3.379	4.9	20.00	−30.11	−0.152
12	2.104	4.950	1.444	2.670	4.793	1.821	2.701	3.896	0.9	8.53	−11.11	−0.061
13	2.230	3.976	1.916	1.727	5.076	1.507	2.827	3.693	1.6	12.18	−12.24	−0.071
14	2.890	5.264	2.481	2.010	5.044	1.256	2.167	3.516	1.2	8.75	−13.87	−0.055
15	2.701	5.296	2.984	2.293	4.070	1.476	1.193	3.630	5.1	23.42	−26.80	−0.173
16	1.130	3.567	1.476	1.507	4.164	2.293	1.067	3.641	4.8	18.73	−29.34	−0.090
17	2.010	4.164	0.910	1.696	5.359	2.010	2.953	3.971	3.5	28.07	−17.32	−0.143
18	0.879	5.076	1.099	2.576	3.693	2.136	2.356	3.933	1.2	9.51	−12.99	−0.046
19	2.921	3.787	1.947	1.067	4.950	1.319	1.601	3.290	3.0	18.99	−19.43	−0.078
20	1.664	4.699	1.601	1.853	4.667	2.199	2.670	3.852	1.7	11.87	−12.61	−0.071
21	2.796	5.359	2.387	2.481	5.421	2.073	0.816	3.990	13.7	42.52	−51.29	−0.242
22	0.910	4.730	2.450	2.104	3.913	1.161	1.821	3.693	2.0	10.60	−16.58	−0.064
23	1.350	4.416	2.419	2.230	4.039	2.513	2.419	4.074	1.5	10.23	−12.44	−0.047
24	1.067	5.233	2.041	0.941	3.976	2.984	0.973	3.997	1416.3	421.82	−409.31	−4.525
25	1.727	3.379	2.576	1.947	3.724	1.539	1.319	3.500	3.3	17.49	−23.41	−0.105
26	1.570	3.819	1.633	1.319	3.473	2.230	2.073	3.521	3.3	17.77	−19.19	−0.055
27	2.324	3.944	0.941	1.570	3.379	2.639	1.664	3.638	2.0	16.11	−18.12	−0.062
28	1.507	3.536	2.104	1.444	4.384	2.890	1.759	4.038	6.2	25.93	−24.97	−0.116
29	2.984	4.887	2.199	1.916	3.881	1.696	1.381	3.425	2.7	15.66	−22.36	−0.073
30	2.764	4.919	1.067	2.513	4.824	0.973	1.130	3.529	11.9	42.82	−68.82	−0.101
31	1.319	4.793	2.733	1.633	3.316	1.916	2.607	3.801	4.4	24.56	−27.48	−0.067
32	1.256	3.756	1.224	2.261	3.347	2.324	1.224	3.725	5.9	29.30	−41.53	−0.060
33	2.199	4.353	1.727	1.036	4.699	1.601	1.036	3.248	4.4	20.62	−28.96	−0.100
34	1.381	3.661	1.790	2.859	4.761	1.727	1.979	4.065	2.0	14.69	−16.89	−0.053
35	2.859	4.101	1.507	1.539	5.453	2.796	2.733	4.220	2.4	17.39	−14.00	−0.091
36	2.544	4.761	2.701	2.953	5.390	2.701	2.261	4.473	1.6	11.38	−13.70	−0.068
37	1.790	5.421	1.759	1.476	3.756	2.419	1.884	3.709	2.6	15.04	−16.35	−0.048
38	1.476	4.227	2.859	0.847	5.296	1.004	1.413	3.590	7.4	27.23	−29.91	−0.117
39	0.847	4.196	2.921	2.136	4.447	2.764	2.104	4.420	5.0	21.79	−21.72	−0.104
40	1.696	4.981	0.816	2.639	4.636	2.576	0.910	4.182	38.1	78.35	−139.71	−0.180
41	2.639	5.107	1.004	1.884	4.133	1.664	2.293	3.490	1.5	13.26	−14.13	−0.060
42	2.827	4.510	1.539	2.890	3.661	2.733	2.324	4.109	1.4	11.04	−13.75	−0.054
43	0.816	4.384	1.664	2.921	4.919	1.130	2.450	4.026	1.2	9.74	−12.73	−0.077
44	1.821	5.170	1.696	1.099	4.353	0.941	2.010	3.167	3.4	19.58	−19.18	−0.086
45	2.073	4.007	2.230	1.130	4.007	2.167	2.576	3.644	4.3	21.49	−21.89	−0.062
46	2.261	4.447	2.356	1.664	3.819	1.979	1.570	3.500	2.2	10.95	−19.12	−0.068
47	1.979	4.604	2.010	2.387	3.536	0.910	2.387	3.491	2.2	13.54	−14.43	−0.063
48	1.413	3.693	2.513	0.973	3.504	2.670	1.727	3.902	19.4	57.55	−48.43	−0.355
49	1.853	4.070	1.130	2.607	4.573	1.947	1.916	3.866	2.2	15.88	−19.15	−0.067
50	1.036	3.347	0.879	0.910	4.321	1.287	1.256	3.154	6.3	28.14	−26.93	−0.206
51	2.450	4.636	2.293	1.224	4.510	2.041	1.790	3.530	2.7	15.14	−17.26	−0.061
52	1.004	5.390	1.161	0.816	3.944	1.099	2.890	3.412	10.0	37.09	−31.38	−0.156
53	0.973	5.044	2.670	2.796	3.599	2.387	0.941	4.250	11.7	37.54	−44.30	−0.261
54	1.947	4.259	1.036	2.733	3.567	1.884	1.947	3.745	2.7	17.85	−23.38	−0.056
55	2.733	3.913	1.979	1.601	4.227	1.036	2.041	3.178	1.8	12.56	−15.06	−0.041
56	2.167	4.667	2.827	1.256	3.630	2.859	1.099	3.950	3100.6	696.71	−692.80	−4.549
57	1.884	5.327	2.764	2.167	5.013	2.481	1.539	4.141	3.0	15.28	−19.99	−0.102
58	2.293	4.290	2.544	1.004	3.787	1.224	2.764	3.279	3.1	17.21	−19.62	−0.080
59	1.539	4.133	1.319	1.193	4.730	1.633	1.444	3.371	3.6	19.62	−21.15	−0.139
60	1.759	5.013	2.639	2.356	4.604	1.067	2.984	3.780	0.9	7.13	−10.33	−0.048
61	1.224	3.599	2.953	2.199	5.107	1.444	0.847	4.060	10.5	35.52	−38.95	−0.262
62	1.287	5.453	2.167	2.827	4.479	2.544	2.544	4.324	1.1	8.24	−11.89	−0.036
63	2.481	3.316	2.073	2.701	3.850	1.350	2.230	3.622	1.6	10.16	−13.80	−0.040
64	2.419	5.484	1.884	2.324	4.259	1.570	2.796	3.690	1.0	8.86	−11.06	−0.034
65	2.576	3.850	0.847	1.821	4.101	2.827	2.859	4.027	2.8	25.72	−16.60	−0.056
66	1.916	3.410	1.381	2.419	4.856	1.381	2.199	3.762	1.2	9.55	−13.81	−0.079
67	1.099	5.139	2.136	2.764	5.327	2.261	2.639	4.398	1.2	7.80	−11.39	−0.056
68	1.193	4.856	1.193	1.790	4.541	1.413	2.921	3.623	2.3	19.47	−14.15	−0.098
69	2.513	3.441	1.256	2.450	5.170	0.847	1.507	3.690	3.9	22.41	−29.89	−0.111
70	2.670	4.541	2.261	1.287	5.264	1.853	0.879	3.529	5.8	22.56	−34.68	−0.117

Table A2 Initial 90 LHS sample data sets

	Design variables							AttributesConstraints				
	x_1	x_2	x_3	y_3	x_4	y_4	y_6	W	D_M	σ_{TM}	σ_{CM}	$R_{\text{BK-MIN}}$
No.	m	m	m	m	m	m	m	ton	mm	MPa	MPa	——
1	1.350	4.217	2.548	2.303	4.706	1.766	1.912	3.943	1.7	11.05	−15.87	−0.067
2	2.157	4.632	1.277	2.254	5.317	0.861	2.401	3.580	1.2	10.48	−12.58	−0.121
3	1.986	3.434	2.059	1.594	4.559	1.497	1.032	3.431	4.4	21.13	−30.13	−0.087
4	1.032	4.437	1.790	2.474	4.339	2.548	2.206	4.138	1.5	8.68	−13.75	−0.042
5	0.959	4.852	1.081	1.570	5.268	1.448	1.888	3.606	2.7	15.67	−16.56	−0.179
6	1.472	4.168	2.279	0.959	4.217	2.010	2.377	3.681	16.9	48.33	−49.40	−0.153
7	2.108	3.508	2.597	2.401	3.312	1.570	0.837	3.574	12.4	39.16	−43.17	−0.245
8	2.694	3.361	1.814	1.668	4.192	1.912	2.670	3.587	1.6	12.55	−12.84	−0.035
9	2.377	3.997	2.792	2.034	4.437	1.643	0.861	3.586	9.1	33.35	−37.79	−0.221
10	0.983	3.606	0.959	1.472	3.581	0.812	2.817	3.246	4.9	29.87	−18.34	−0.082
11	1.814	4.559	2.866	0.983	3.508	2.841	1.301	4.017	8.9	35.71	−28.39	−0.198
12	0.910	4.901	1.961	2.792	4.657	2.132	1.863	4.196	2.3	15.43	−17.82	−0.053
13	1.106	5.439	1.546	2.059	4.974	1.081	1.741	3.600	1.8	10.82	−17.59	−0.094
14	2.059	4.266	0.934	1.203	3.434	1.472	2.303	3.225	4.3	30.68	−20.13	−0.060
15	2.817	3.557	1.179	2.768	3.777	2.621	2.328	4.021	1.4	10.66	−13.75	−0.049
16	2.523	4.828	1.399	0.812	3.948	1.692	1.961	3.276	5.4	28.33	−19.80	−0.071
17	1.497	3.948	1.448	2.914	4.803	1.203	2.646	3.915	1.0	7.93	−11.53	−0.079
18	2.034	5.341	1.570	2.352	5.146	1.937	2.621	3.921	1.1	9.42	−11.61	−0.072
19	1.790	4.583	1.692	1.619	4.828	2.963	1.717	4.055	2.8	14.94	−17.50	−0.102
20	2.132	3.972	2.646	2.939	3.654	1.154	0.812	3.775	18.8	46.69	−59.40	−0.348
21	2.206	3.483	1.374	2.548	5.488	0.886	2.426	3.884	1.1	9.08	−12.41	−0.116
22	0.861	4.363	1.326	2.719	3.606	1.228	1.277	3.734	8.7	33.99	−51.10	−0.085
23	1.374	4.461	1.350	2.841	4.999	2.157	1.692	4.179	4.0	22.24	−30.18	−0.073
24	1.741	3.386	1.619	2.377	5.097	1.008	1.130	3.765	6.7	30.79	−40.90	−0.089
25	1.570	3.581	2.181	2.743	5.366	2.279	2.230	4.401	1.5	10.43	−13.82	−0.052
26	1.154	5.390	1.032	1.448	3.801	1.961	2.108	3.556	2.9	18.20	−16.89	−0.085
27	2.303	4.608	0.837	1.154	4.510	1.374	1.472	3.173	3.3	22.37	−21.80	−0.129
28	1.766	3.752	1.497	0.934	4.632	2.254	2.034	3.652	6.0	24.11	−21.93	−0.162
29	1.912	5.366	0.886	2.450	4.608	1.277	1.081	3.649	15.9	50.92	−85.76	−0.098
30	1.961	4.706	2.230	2.328	5.219	1.179	1.399	3.704	3.3	19.29	−23.23	−0.086
31	0.934	3.679	2.108	1.814	4.754	2.646	2.474	4.249	3.5	18.46	−18.38	−0.090
32	2.083	4.143	2.621	1.374	4.779	2.352	1.619	3.836	4.0	19.55	−19.03	−0.082
33	1.228	5.219	1.912	2.597	4.461	0.934	1.766	3.741	2.5	15.38	−18.52	−0.055
34	1.546	4.021	1.472	2.890	3.557	2.059	1.668	3.898	4.0	22.37	−29.86	−0.054
35	1.130	5.268	2.988	1.301	5.072	0.959	2.181	3.729	2.1	11.58	−15.37	−0.089
36	2.328	4.486	2.328	2.621	3.997	2.890	0.983	4.149	9.9	35.41	−42.95	−0.192
37	1.057	3.410	2.841	1.130	3.899	2.230	2.450	4.006	20.5	56.91	−49.33	−0.410
38	2.254	3.337	1.228	1.252	3.728	2.719	2.279	3.757	3.4	22.02	−17.16	−0.063
39	2.914	3.801	2.743	2.988	3.337	1.888	2.548	3.926	1.6	9.35	−12.16	−0.059
40	1.203	4.412	1.252	1.277	4.926	2.694	2.719	4.083	6.0	25.42	−20.43	−0.202
41	2.572	3.923	0.812	1.839	4.877	1.814	1.594	3.601	2.0	14.11	−19.66	−0.099
42	1.179	4.534	2.254	1.179	4.314	2.866	2.743	4.214	92.2	105.37	−104.33	−0.471
43	2.768	5.146	2.474	2.279	3.752	2.181	1.154	3.750	5.2	24.83	−28.56	−0.134
44	2.010	3.703	1.766	2.963	3.459	2.597	2.523	4.074	1.2	9.27	−12.25	−0.059
45	1.839	4.730	2.377	1.986	4.583	1.594	2.866	3.738	1.1	9.12	−11.16	−0.046
46	0.886	5.170	1.008	2.572	3.923	2.670	2.963	4.187	1.1	9.70	−10.86	−0.054
47	1.081	5.048	0.983	2.817	5.048	2.817	2.694	4.445	1.2	8.90	−11.22	−0.092
48	1.252	4.779	1.643	0.837	4.950	1.863	1.106	3.480	11.5	33.99	−34.73	−0.265
49	1.399	4.681	1.839	1.423	4.046	1.423	2.841	3.503	2.5	16.28	−16.17	−0.050
50	2.230	3.777	2.426	1.546	5.121	2.792	0.934	4.068	6.8	24.38	−33.13	−0.132
51	1.521	3.532	1.203	2.426	3.361	2.303	2.010	3.750	1.6	11.34	−15.79	−0.059
52	2.841	3.728	1.668	1.741	3.850	1.057	1.986	3.141	1.9	12.44	−15.27	−0.041
53	2.939	5.072	1.986	1.717	3.874	1.106	1.839	3.228	1.9	11.69	−16.34	−0.041
54	1.423	4.950	2.034	2.010	4.486	1.668	1.643	3.665	2.0	11.80	−18.55	−0.050
55	1.008	4.314	1.057	2.646	4.852	1.986	0.959	4.080	22.5	59.15	−99.86	−0.149
56	2.548	4.754	1.594	2.083	3.679	2.328	2.988	3.816	1.3	11.40	−11.55	−0.049
57	2.792	5.121	1.863	1.008	5.463	1.399	1.057	3.304	4.4	20.52	−28.39	−0.121
58	0.837	5.463	1.423	2.230	4.168	1.032	1.374	3.625	3.9	21.51	−26.27	−0.065
59	1.717	4.388	1.741	1.081	3.410	2.988	1.448	3.804	8.7	32.44	−30.86	−0.126
60	2.499	4.926	0.861	1.228	3.703	2.083	2.254	3.527	4.3	31.72	−20.70	−0.060
61	2.719	3.850	2.157	1.692	3.386	2.743	1.497	3.765	2.6	11.79	−20.12	−0.063
62	1.619	4.046	2.890	0.910	5.194	2.401	1.179	4.055	761.9	299.57	−298.22	−1.430
63	2.988	4.119	1.301	1.961	4.290	2.914	1.350	3.955	3.1	18.52	−24.55	−0.042
64	2.743	3.899	2.939	2.206	4.363	2.450	2.352	3.978	1.3	7.53	−12.77	−0.070
65	2.474	4.999	2.132	1.032	3.483	1.741	2.792	3.469	3.5	18.74	−16.43	−0.068
66	1.692	5.292	2.963	1.399	5.023	2.939	1.937	4.344	33.1	61.78	−59.92	−0.252
67	2.352	3.874	2.206	1.350	5.390	1.350	2.083	3.593	2.3	14.05	−15.18	−0.087
68	1.277	4.192	1.937	1.057	4.021	2.768	1.326	3.826	28.4	60.04	−59.31	−0.275
69	2.426	4.070	2.010	0.886	4.119	1.546	2.914	3.384	4.6	22.52	−18.43	−0.076
70	1.863	4.877	2.914	1.863	3.972	2.499	0.910	3.927	9.6	29.30	−34.69	−0.208

(continued)

Table A2 Initial 90 LHS sample data sets (Continued)

	Design variables							AttributesConstraints				
	x_1	x_2	x_3	y_3	x_4	y_4	y_6	W	D_M	σ_{TM}	σ_{CM}	R_{BK-MIN}
No.	m	m	m	m	m	m	m	ton	mm	MPa	MPa	——
71	2.866	3.459	2.572	2.157	5.292	1.301	2.059	3.768	1.3	8.65	−14.58	−0.061
72	1.448	3.654	0.910	1.106	3.630	1.521	1.570	3.113	3.7	23.01	−21.45	−0.093
73	2.181	4.290	2.083	2.108	4.901	2.034	1.423	3.793	2.8	16.78	−22.19	−0.068
74	1.326	5.194	2.670	1.912	5.341	2.572	1.521	4.286	4.0	16.05	−19.93	−0.092
75	1.888	4.803	2.719	1.790	4.730	0.983	1.252	3.448	3.4	17.72	−24.43	−0.119
76	1.937	5.097	1.154	1.643	4.266	1.326	1.228	3.269	3.0	16.85	−25.96	−0.081
77	1.301	4.241	2.499	2.181	5.170	2.426	2.939	4.358	1.6	11.37	−11.39	−0.052
78	2.890	5.488	2.817	1.497	5.439	1.619	2.157	3.640	1.7	10.63	−14.28	−0.076
79	0.812	3.312	2.352	2.523	3.826	1.130	2.890	3.876	1.4	9.62	−10.84	−0.041
80	2.597	4.657	2.523	2.694	3.532	2.474	2.572	4.018	1.0	7.62	−11.68	−0.055
81	2.646	4.339	1.130	0.861	4.143	0.837	1.790	2.949	6.0	28.28	−21.70	−0.114
82	1.643	5.023	1.717	1.521	4.681	1.839	2.768	3.694	2.5	15.74	−14.48	−0.082
83	2.450	5.243	2.694	2.132	5.414	1.790	1.008	3.833	6.7	28.07	−32.25	−0.175
84	1.594	3.630	2.450	2.866	4.094	1.252	0.886	3.861	14.8	42.58	−54.73	−0.275
85	2.621	4.510	2.768	1.766	5.243	2.377	1.203	3.935	4.1	18.73	−25.45	−0.129
86	2.401	5.317	1.888	1.326	4.388	2.108	2.597	3.662	2.5	15.55	−13.93	−0.055
87	2.279	4.974	2.303	2.499	4.534	1.717	1.546	3.756	2.9	17.72	−21.00	−0.083
88	2.963	4.094	1.106	2.670	4.412	2.206	2.499	3.917	1.0	9.16	−12.15	−0.045
89	2.670	5.414	2.401	1.937	4.241	0.910	2.132	3.373	1.6	9.66	−14.12	−0.052
90	1.668	3.826	1.521	1.888	4.070	2.523	1.814	3.784	1.9	11.86	−16.56	−0.051

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